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ASYMPTOTIC THEORY OF THE FLOW AROUND AN OBSTACLE BY A SONIC FLOW

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The first investigation of the problem of the flow around an obstacle by a gas flow whose velocity is equal to the speed of sound at infinity was carried out in [1, 2], where it is shown in particular that the principal term of the appropriate asymptotic expansion is a self-similar solution of Tricomi's equation, to which the problem reduces in the first approximation upon a hodographic investigation. The requirement that the stream function be analytic as a function of the hodographic variables on the limiting characteristic was an important condition determining the selection of the self-similarity exponent n (xy^{-n} is an invariant of the self-similar solution). The analytic nature of the velocity field everywhere in the flow above the shock waves, which arise from necessity upon flow around an obstacle, follows from this condition. The latter was found in [3], where one of the branches of the solution obtained in [1] was used in the region behind the shock waves. The principal and subsequent terms of the asymptotic expansion describing a sonic flow far from an obstacle were discussed in [4], where the author restricted himself to Tricomi's equation. Each term of the series constructed in [4] contains an arbitrary coefficient (we will call it a shape parameter) which is not determined within the framework of a local investigation, and consideration of the problem of flow around a given obstacle as a whole is necessary in order to determine these shape parameters. It follows from the results of [4] that the problem of higher approximations to the solution of [1] coincides with the problem of constructing a flow in the neighborhood of the center of a Laval nozzle with an analytic velocity distribution along the longitudinal axis (a Meyer-type flow). Along with the Meyer-type flow in the vicinity of the nozzle center, which corresponds to a self-similarity exponent n = 2, two other types of flow are asymptotically possible with n=3 and 11, given in [5]. The appropriate solutions are written out in algebraic functions in [6]. The results of [5] show that the condition that the velocity vector be analytic on the limiting characteristic in the flow plane is broader than the condition that the stream function be analytic as a function of the hodographic variables, which is employed in [1, 2, 4]. Therefore, the necessity has arisen of reconsidering the problem of higher approximations for the obstacle solution of F. I. Frankl'. It has proved possible for the region in front of the shock waves to use a series which is more general than in [4], which implies the inclusion of an additional set of shape parameters. The solution is given in the hodograph plane in the form of the sum of two terms; the series discussed in [4] corresponds to the first one, and the series generated by the self-similar solution with n = 3 or with n = 11 corresponds to the second one.

1. Two dimensional irrotational flows of an ideal perfect gas are described in the transonic approximation by the equations [7]

$$-uu_x + v_y = 0, \ u_y - v_x = 0, \tag{1.1}$$

where x and y are the reduced Cartesian coordinates and u and v are the dimensionless components of perturbations of a uniform sonic flow.

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System (1.1) reduces on the hodograph plane to Tricomi's equation

$$-uy_{vv} + y_{uu} = 0. \tag{1.2}$$

The coordinate x is found from the relationship

$$dx = uy_v du + y_u dv. \tag{1.3}$$

Equation (1.2) possesses the class of self-similar solutions

$$y = v^\lambda f_\lambda(t),\ t = v/\sqrt{v^2 - (4/9)u^3}, \tag{1.4}$$

where λ is a parameter uniquely associated with the self-similarity exponent $n = (\lambda + 1/3)/\lambda$ and t is the self-similar variable.

The function $f_\lambda(t)$ satisfies the ordinary differential equation

$$t^2(1-t^2)f_\lambda'' + t[2\lambda - (2\lambda + 4/3)t^2]f_\lambda' + \lambda(\lambda - 1)f_\lambda = 0, \tag{1.5}$$

whose general solution is expressed in terms of the hypergeometric functions (A_λ and B_λ are arbitrary constants)

$$f_\lambda = A_\lambda t^{-\lambda} F(-\lambda/2,\ \lambda/2 + 1/6,\ 1/2;\ t^2) + B_\lambda t^{1-\lambda} F(-\lambda/2 + 1/2,\ \lambda/2 + 2/3,\ 3/2;\ t^2). \tag{1.6}$$

If $A_\lambda = 0$, then (1.4) and (1.6) determine a flow which is symmetric with respect to the x axis.

From (1.3) we find

$$x = v^{\lambda+1/3} g_\lambda(t),\ g_\lambda = (3/2)^{1/3}(\lambda + 1/3)^{-1}(t^{-2} - 1)^{2/3}(\lambda f_\lambda + t f_\lambda'). \tag{1.7}$$

2. The flows described by the solutions of Eq. (1.2), in which the condition of analyticity of the velocity field is satisfied on the limiting characteristic which is the boundary of the transonic zone, are called natural or N flows. The condition of naturalness determines a wider class than the condition of regularity of y on the line $t = \infty$ in the hodograph plane. They were investigated in [8] in a class of self-similar N flows.

At first we will restrict ourselves to the study of N flows which are symmetric with respect to the x axis (NS flows). Below are considered non-self-similar solutions of Tricomi's equation which are series in the self-similar components with the principal term describing an NS flow without limiting lines in the transonic zone.

We will discuss the problem of higher approximations to the solution of [1]. We present the desired solution of Eq. (1.2) in the form

$$y = v^{-5/3} f_{-5/3} + v^\lambda f_\lambda. \tag{2.1}$$

Let us determine the values of the exponent λ appearing in the second term for which (2.1) determines an NS flow. For definiteness, we will consider the region of the hodograph bounded by the negative semiaxis of u and the limiting characteristic $v = 2/3 u^{3/2}$, with $v > 0$. We write the general integral of Eq. (1.5) in the form

$$f_\lambda = C_\lambda F(-\lambda/2,\ -\lambda/2 + 1/2,\ -\lambda + 5/6;\ t^{-2}) + D_\lambda t^\nu F(\lambda/2 + 2/3,\ \lambda/2 + 1/6,\ \lambda + 7/6;\ t^{-2}),\ \nu = -2\lambda - 1/3, \tag{2.2}$$

where C_λ and D_λ are constants associated with A_λ and B_λ by a nondegenerate linear transformation, and

$$\nu \neq 0, \pm 2, \pm 4, \ldots. \tag{2.3}$$

If ν is an even number, then (2.2) should be replaced by an expression containing logarithmic terms.

In [1, 4] the exponents λ are found from the symmetry condition $A_\lambda = 0$ and the regularity condition of the solution (2.1) in the hodograph plane $D_\lambda = 0$. In order to discover exponents λ which are different from the ones indicated in [1, 4], we will consider the case $D_\lambda \neq 0$.

Using (1.3) and (2.1), we find

$$x = v^{-4/3} g_{-5/3} + v^{\lambda+1/3} g_\lambda. \tag{2.4}$$

From (2.1) and (2.4) we obtain the expansion $\zeta = xy^{-4/5}$ in powers of y with coefficients depending on t:

$$\begin{gathered}\zeta = \zeta_0(t) + y^h \zeta_1(t) + y^{2h}\zeta_2(t) + \ldots,\ h = -3\lambda/5 - 1,\\ \zeta_0 = g_{-5/3} f_{-5/3}^{-4/5},\ \zeta_1 = -(4/5) f_\lambda f_{-5/3}^{-1-h} + g_\lambda f_{-5/3}^{-4/5-h},\\ \zeta_2 = (9/5 + 2h)(2/5) f_\lambda^2 f_{-5/3}^{-2-2h} - (4/5 + h) g_\lambda f_\lambda f_{-5/3}^{-9/5-2h}, \ldots.\end{gathered} \tag{2.5}$$

We will expand the function ζ_i into series as $t \to \infty$,

$$\zeta_0 = \zeta_{00} + \zeta_{02} t^{-2} + \ldots,\ \zeta_i = \zeta_{i0} + \zeta_{i1} t^\nu + \zeta_{i2} t^{-2} + \ldots, \tag{2.6}$$

where ζ_{ij} are constants expressed in terms of $C_{-5/3}$, λ, C_λ, D_λ.

We note that ζ_0 is an analytic function of the variable t^{-2}, since $D_{-5/3} = 0$.

We investigate the case $-2<\nu<0(-1/6<\lambda<5/6)$. Using (2.6), we rewrite (2.5) in the form

$$\begin{aligned}\zeta &= Z_0(y) + Z_1(y)t^\nu + Z_2(y)t^{-2} + \ldots,\\ Z_0 &= \zeta_{00} + \zeta_{10}y^h + \zeta_{20}y^{2h} + \ldots,\\ Z_1 &= \zeta_{11}y^h + \zeta_{21}y^{2h} + \ldots,\\ Z_2 &= \zeta_{02} + \zeta_{12}y^h + \zeta_{22}y^{2h} + \ldots.\end{aligned} \tag{2.7}$$

On the limiting characteristic $t=\infty$; therefore, $\zeta=Z_0$ is the equation of the limiting characteristic in the x, y plane. Inverting (2.7), we find the expansion of the quantity t^{-2} in the vicinity of $\zeta=Z_0$:

$$t^{-2} = Z_1^{2/\nu}(\zeta - Z_0)^{-2/\nu} + \ldots. \tag{2.8}$$

On the other hand,

$$t^{-2} = 1 - 4u^3/(9v^2). \tag{2.9}$$

Let u, v be analytic functions of x, y on the limiting characteristic. Then it follows from (2.8) and (2.9) that the exponent $-2/\nu$ should be a natural number:

$$-2/\nu = i,\ \lambda = 1/i - 1/6,\ i = 1,\ 2, \ldots$$

Even values of i correspond to flows for which the limiting characteristic is a branching line in the hodograph plane. Since the basic solution with $\lambda=-5/3$ does not possess this property, the continuation beyond the limiting characteristic is not determined for the corresponding sum. The value $i=1$ is unsuitable, since the condition (2.3) is violated. With $i=3$ we obtain $\lambda=1/6$; i.e., the correction in (2.1) corresponds to the second asymptotic type of flow in a two-dimensional nozzle [5]. Let $i=5$; then $\lambda=1/30$, and the correction corresponds to the third asymptotic type of flow in a Laval nozzle [5].

If $i=7, 9, 11, \ldots$, then $\lambda=-1/42, -1/18, -5/66, \ldots$. Nozzle flows with a straight sonic line which were studied in [9] correspond to these values. The absence of the region of the hodograph between the sonic line and the limiting characteristic in these flows prevents their use as a term in (2.1).

Now let $-\infty<\nu<-2(5/6<\lambda<\infty)$. Inverting (2.7), we obtain

$$t^{-2} = Z_2^{-1}(\zeta - Z_0) - Z_1 Z_2^{\nu/2-1}(\zeta - Z_0)^{-\nu/2} + \ldots$$

If the velocity field is analytic on the limiting characteristic, the exponent $-\nu/2$ should be a natural number; however, the condition (2.3) is violated in this case.

Therefore, we obtain that (2.1) determines an N flow if $\lambda=1/6$ and $1/30$.

3. Higher approximations for self-similar solutions (1.4) with the exponents $\lambda=-5/3$, $1/6$, and $1/30$ are constructed, each of which contains a shape parameter. Thus the NS flow described by the series

$$y_1 = v^{\lambda_{i1}} f_{\lambda_{i1}},\ \lambda_{i1} = -5/3 + 2i/3 \tag{3.1}$$

was investigated in [4]. Here summation is carried out over the repeated subscript i, which takes integral negative values not equal to 1 and 2. Since $D_{\lambda_{i1}}=0$ in (2.2), which corresponds to (3.1), the coefficients $f_{\lambda i1}$ (and also $g_{\lambda i1}$) are analytic functions of the variable $t^{-2}(t\to\infty)$.

A flow in a doubly symmetric Laval nozzle described by the solution

$$y_2 = v^{\lambda_{j2}} f_{\lambda_{j2}},\ \lambda_{j2} = 1/6 + j/3 \tag{3.2}$$

was investigated in [10], where j is an integral nonnegative summation index not equal to 2, 5, 8, 11, ..., which corresponds to the condition (2.3). We note that $\nu(\lambda_{j2})=-2(j+1)/3$; therefore, $f_{\lambda_{j2}}$, and also $g_{\lambda_{j2}}$, are analytic functions of the variable $\tau=t^{-2/3}$ $(\tau\to 0)$.

We will consider the solution of Eq. (1.2)

$$y_3 = v^{\lambda_{k3}} f_{\lambda_{k3}},\ \lambda_{k3} = 1/30 + k/5, \tag{3.3}$$

where k is an integral nonnegative summation index not equal to 4, 9, 14, ... Since $\nu(\lambda_{k3})=-2(k+1)/5$, $f_{\lambda_{k3}}$ and $g_{\lambda_{k3}}$ are analytic functions of the variable $\tau_1=t^{-2/5}$. It is shown in [10] that (3.2) and (3.3) determine NS flows in a particular case.

4. Using the linearity of Tricomi's equation, we sum up the solutions y_1 and y_2, obtaining

$$y = v^{-5/3}f_{-5/3} + v^{1/6}f_{1/6} + v^{1/3}f_{1/3} + v^{1/2}f_{1/2} + \ldots \tag{4.1}$$

The coefficients $f_{1/6}$, $f_{1/2}$, ... depend parametrically on t [6, 10]:

$$\begin{aligned}t &= 2\cdot 3^{-3/4}(s^5 + s)(-s^4 + 2\sqrt{3}s^2 + 1)^{-3/2},\\ f_{1/6} &= H_0(s + E_0)(s^5 + s)^{-1/6},\\ f_{1/2} &= H_1(s^3 - \sqrt{3}E_1 s^2 + \sqrt{3}s + E_1)(s^5 + s)^{-1/2}, \ldots,\end{aligned}$$

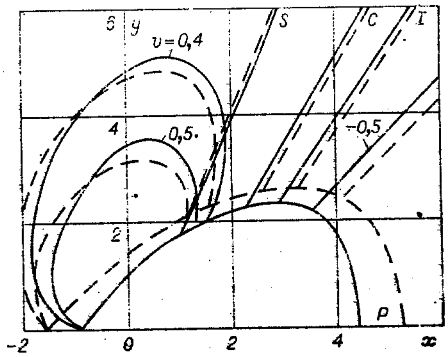

Fig. 1

Here s is a real parameter which varies within the limits $0 < s < \infty$; H_i and E_i are arbitrary constants, and $E_0 = E_1 = \ldots = 0$ for symmetric flows.

The functions $f_{-5/3}$, $f_{1/3}$, ... are of the form

$$f_{-5/3} = (9/16)B_{-5/3}t^{5/3}[(1-t)^{1/3}(1/3+t) - (1+t)^{1/3}(1/3-t)],$$

$$f_{1/3} = (3/2)B_{1/3}t^{-1/3}[(1+t)^{1/3} - (1-t)^{1/3}].$$

We will show that (4.1) determines an NS flow. Using (1.3) and (1.7), we find

$$x = v^{-4/3}g_{-5/3} + v^{1/2}g_{1/6} + v^{2/3}g_{1/3} + v^{5/6}g_{1/2} + \ldots \tag{4.2}$$

The coefficients $f_{-5/3}$, $g_{-5/3}$, $f_{1/6}$, $f_{1/6}$, ... are analytic functions of the variable τ introduced in Sec. 3. Then (4.1) and (4.2) determine $x(v, \tau)$ and $y(v, \tau)$ as analytic functions in the vicinity of the limiting characteristic $\tau = 0$ $(v > 0)$. If the Jacobian of the transformation $J = \partial(x, y)/\partial(v, \tau)$ is different from zero in the vicinity of $\tau = 0$, then it is possible to invert (4.1) and (4.2). The inverse functions $v(x, y)$ and $\tau(x, y)$ are analytic on the limiting characteristic. The function $u(x, y)$ will possess the same property, since

$$u = (3/2)^{2/3}v^{2/3}(1-\tau^3)^{1/3}. \tag{4.3}$$

Let us determine the sufficient conditions for which $J \neq 0$ in the vicinity of $\tau = 0$ in the case of small values of v. Let us represent J in the form

$$J = -(3/2)^{2/3}v^{2/3}(\tau - \tau^4)^{-1}(x_v - \sqrt{u}\, y_v)(x_v + \sqrt{u} y_v).$$

Expanding x_v, y_v in powers of τ with v fixed, we obtain

$$J = v^{-1/2}\, 2(3/2)^{1/3}D_{1/6}[-(5/3)\; C_{-5/3}v^{-5/3} + (1/6)\, C_{1/6}v^{1/6} + \ldots] + O(\tau).$$

If $D_{1/6} \neq 0$, and in addition

$$\operatorname{sgn}\,(C_{-5/3}C_{1/6}) = -1, \tag{4.4}$$

$J \neq 0$ in the vicinity of the limiting characteristic for sufficiently small fixed positive values of v.

The results of the calculation of the characteristic lines in the case of flow around an obstacle are given in Fig. 1. The flow (4.1) in which the first two (three) terms are kept is shown by solid (dashed) lines; S denotes the sonic line, C denotes the limiting characteristic, I denotes the zero-inclination line of the velocity vector, and P denotes the zero line of the flow calculated by integration of the equation $dy/dx = v$. Several lines of $v = \text{const}$ are also shown. It was assumed in the calculations that

$$B_{-5/3} = 16/9,\ B_{1/3} = -4^{-1}(2/3)^{2/3},$$
$$H_0 = -2^{1/6}3^{-1/12}(2+\sqrt{3})^{-1/4},\ E_0 = 0. \tag{4.5}$$

5. We will construct a solution of the system (1.1), which corresponds to (4.1), with the help of an expansion of the desired functions into series in self-similar components on the x, y plane. We will first use coordinate expansions in powers of y with ζ fixed:

$$u = y^{-2/5+k_i}\,U_i(\zeta),\ v = y^{-3/5+k_i}\,V_i(\zeta),\ \zeta = xy^{-4/5},$$
$$k_0 = 0,\ k_1 = -11/10,\ k_2 = -6/5,\ k_3 = -13/10,\ k_4 = -8/5, \tag{5.1}$$
$$k_5 = -17/10,\ k_6 = -19/10,\ k_7 = -2,\ k_8 = -11/5, \ldots$$

The values of k_i are determined by the corresponding exponents of the powers in (4.1). The functions U_i and V_i satisfy a system of ordinary differential equations. This system has a singular point ζ_c, which is determinable from the condition $4U_0^3(\zeta_c) = 9V_0^2(\zeta_c)$. The generalized parabola $\zeta = \zeta_c$ is the limiting characteristic of the system (1.1) for the self-similar solution

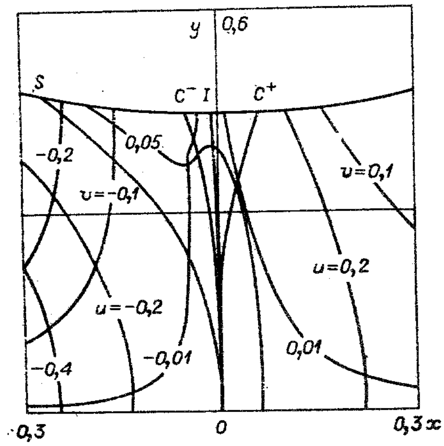

Fig. 2

$$u = y^{-2/5}U_0(\zeta),\ v = y^{-3/5}V_0(\zeta). \tag{5.2}$$

If all the functions U_i and V_i are analytic at the point ζ_c, then (5.1) determines the solution of the system (1.1) which is analytic on the limiting characteristic.

An investigation of the higher approximations (5.1) shows that the functions U_i, V_i, ... are nonanalytic at the point ζ_c. However, one should not interpret this fact as proof of nonanalyticity of the velocity field determined by the hodographic solution (4.1). The cause of this apparent contradiction is explained by the fact of the nonuniform validity of the asymptotic expansion (5.1), which is valid near the limiting characteristic, whereas (4.1) is uniformly valid everywhere. Outside the vicinity of the limiting characteristic the expansions (4.1) and (5.1) are equivalent. The singular nature of the expansion (5.1) near $\zeta = \zeta_c$ arises due to the fact that the system (1.1) is nonlinear, and upon perturbation of the self-similar solution (5.2) the limiting characteristic deviates from the generalized parabola $\zeta = \zeta_c$, which is not taken into consideration in this expansion.

In order to construct a uniformly valid expansion, let us use the method of deformed coordinates, expanding the variable ζ also together with u and v into a series in powers of y. The coefficients of the expansions can be conveniently assumed to be dependent on t:

$$u = y^{-2/5+k_i}u_i(t),\ v = y^{-3/5+k_i}v_i(t); \tag{5.3}$$

$$\zeta = y^{k_i}\zeta_i(t),\quad i = 0, 1, \ldots . \tag{5.4}$$

From (4.1), (4.3), and (5.3) we obtain

$$u_0 = (3/2)^{2/3}(1-\tau^3)^{1/3}f_{-5/3}^{2/5},\ u_i = (2/5)u_0Y_i,\quad i = 1, \ldots, 7,$$
$$u_8 = (7/25)u_0Y_1^2 + (3/2)u_1Y_1, \ldots, Y_1 = f_{1/6}f_{-5/3}^{1/10}, Y_2 = f_{1/3}f_{-5/3}^{1/5}, \ldots \tag{5.5}$$

Analogous formulas are valid for v_i.

Making use of (4.1), (4.2), and (5.4), we find

$$\zeta_0 = g_{-5/3}f_{-5/3}^{-4/5},\ \zeta_i = -(4/5)\zeta_0Y_i + X_i,\quad i = 1, \ldots, 7,$$
$$\zeta_8 = -(4/25)\zeta_0Y_1^2 + (3/10)X_1Y_1, \ldots, X_1 = g_{1/6}f_{-5/3}^{3/10}, X_2 = g_{1/3}f_{-5/3}^{2/5}, \ldots .$$

Let us convince ourselves of the fact that (5.3) and (5.4) determine a velocity field which is analytic on the limiting characteristic. Actually, ζ_i are analytic functions of τ:

$$\zeta_i = \zeta_{ij}\tau^j,\ i, j = 0, 1, \ldots .$$

Then it is possible to rewrite (5.4) in the form of a power series in τ:

$$\zeta = Z_j(y)\tau^j,\quad j = 0, 1, \ldots, Z_j(y) = \zeta_{ij}y^{k_i},\quad i = 0, 1, \ldots, \tag{5.6}$$

and $\zeta = Z_0(y)$ is the equation of the limiting characteristic. Inverting the series (5.6), we obtain τ as an analytic function of the variables ζ and y (or x and y) on the limiting characteristic. According to the construction of (5.5), u_i and v_i are analytic functions of τ; therefore, the velocity field (5.3) and (5.4) satisfies the analyticity condition.

The symmetry condition is also satisfied if the corresponding partial integrals ($A_{\lambda i1} = A_{\lambda i2} =$) are taken on the hodograph plane.

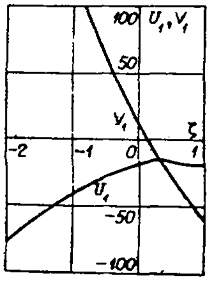

Fig. 3

Since the system of coordinates t, y is used in (5.3) and (5.4), it is necessary that the Jacobian of the transformation $I = \partial(\zeta, y)/\partial(t, y)$ be different from zero. One can show that $I \neq 0$ everywhere in the transonic zone for sufficiently large y if the condition (4.4) is satisfied.

We note that in this construction the hodographic variable t plays the role of the optimal coordinate defined in [11], since the use of it immediately results in a uniformly valid expansion.

6. Now let us consider the expansion (4.1), in which we set $f \equiv 0$. Then the chief term of the expansion will be the nozzle solution with $\lambda = {}^1/_6$:

$$y = v^{1/6} f_{1/6} + v^{1/3} f_{1/3} + v^{1/2} f_{1/2} + \ldots \tag{6.1}$$

Repeating the discussions of the preceding sections, one can prove that (6.1) determines an N flow. The results of the calculation of a nozzle according to the solution (6.1), in which the first two terms are kept, is given in Fig. 2. The flow is analytic on the incoming limiting characteristic C^-, and it continues with a weak discontinuity beyond the characteristic C^+, which originates from the nozzle center in order to avoid the appearance of a limiting line between the characteristic C^+ and the semiaxis $x > 0$. The second term is appreciable everywhere, and the continuation with a weak discontinuity constructed in [5] was used for the first term.

The values of the arbitrary constants are given by Eqs. (4.5).

The corresponding solution of the system (1.1) in the x, y plane can be written by using the method of expansion into a series in self-similar components with ζ fixed:

$$u = y^{4+i} U_i(\zeta), \; v = y^{6+i} V_i(\zeta), \; \zeta = xy^{-3}. \tag{6.2}$$

The representatives U_i and V_i are determined by the formulas given in [6, 10] and are analytic functions at the point ζ_c. Deformation of the variable ζ is not required in this case.

Plots of the functions $U_1(\zeta)$ and $V_1(\zeta)$ are given in Fig. 3. If one sets

$$U_1 = V_1 = U_3 = V_3 = \ldots = 0,$$

then the series (6.2) will contain as a particular case the flow in a doubly symmetric nozzle constructed in [10].

7. Let us consider the solution of Tricomi's equation:

$$y = y_1 + y_3, \; A_{\lambda_{k1}} = A_{\lambda_{k3}} = 0, \; B_{1/30} \neq 0. \tag{7.1}$$

Repeating the discussions of Secs. 2-4, one can show that (7.1) determines an NS flow.

Let $B_{-5/3} \neq 0$; then (7.1) describes flow far from an obstacle. The equivalent uniformly valid expansion in the physical plane is represented in deformed form as

$$u = y^{-2/5+k_i} u_i(t), \; v = y^{-3/5+k_i} v_i(t), \; \zeta = y^{k_i} \zeta_i(t),$$

$$\zeta = xy^{-4/5}, \; k_0 = 0, \; k_1 = -51/50, \; k_2 = -57/50, \; \ldots.$$

If $B_{-5/3} = 0$, then (7.1) determines N flows of a gas in two-dimensional nozzles of the third asymptotic type according to [5], which are characterized by the origin of a shock wave at the center. Here only the inlet part of the flow in such a nozzle is discussed. The appropriate expansion in the x, y plane with ζ fixed turns out to be uniformly valid and has the form

$$u = y^{20+k_i} U_i(\zeta), \; v = y^{39+k_i} V_i(\zeta), \; \zeta = xy^{-11},$$
$$k_0 = 0, \; k_1 = 6, \; k_2 = 9, \; k_3 = 12, \ldots,$$

where U_i and V_i are given by formulas given in [6, 10].

8. We note that the solutions

$$y = y_2 + y_3,\ y = y_1 + y_2 + y_3$$

do not determine N flows. One can convince oneself of this fact by considering a particular example. Let us investigate a flow for which

$$y = v^{1/30}f_{1/30} + v^{1/6}f_{1/6},\ x = v^{11/30}g_{1/30} + v^{1/2}g_{1/6}. \tag{8.1}$$

If one fixes v ($v = v_0$), then one can treat (8.1) as the equation of the equal-slope line in the flow plane, which is written in terms of the parameter t. One can show that the curve $v = v_0$ is nonanalytic at the intersection point with the limiting characteristic; i.e., a weak discontinuity exists in the flow (8.1).

The investigation carried out above shows that the series (3.1) used to describe the flow far from an obstacle can be supplemented with new terms containing shape parameters. These are terms generated either by the self-similar solution with $\lambda = 1/6$ (series (3.2)) or by the solution with $\lambda = 1/30$ (series (3.3)). The solution should be represented in deformed form in the physical plane. Since the usual method of expansion in self-similar components was employed in [12], additional terms have not been discovered.

It has been noted in [10, 13] that the symmetry condition for the solutions y_2 and y_3 is not significant; therefore, (4.1) and (7.1) permit constructing flows far from an obstacle, as well as in Laval nozzles, which are asymmetrical with respect to the x axis.

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